

MR. LEVESQUE AP CALCULUS SUMMER WORK 2020

Welcome to the first step in attaining a score of 5 on your 2021 AP Calculus AB exam. Step one is to make sure you all have a solid mastery of trigonometry.

Show all work and answer all questions completely. You may review notes and/or textbooks but, of course, online sites such as Mathway or Symbolab which solve the problem for you are not to be used. You may not use a calculator for Part 1.

On the first day of class I will hand out the solution key and you will correct your work that night at home. All questions will be reviewed in class the next day.

This work should not take more than 6-8 hours to complete. Finish your work early in the summer so you can enjoy peace and relaxation throughout July and August. Feel free to email me at: rjl@fenwick.org if you have any questions.

θ degrees	30°	45°	60°
θ radians	$\frac{\pi}{6}$	$\frac{\pi}{4}$	$\frac{\pi}{3}$
$\sin \theta$	$\frac{1}{2}$	$\frac{\sqrt{2}}{2}$	$\frac{\sqrt{3}}{2}$
$\cos \theta$	$\frac{\sqrt{3}}{2}$	$\frac{\sqrt{2}}{2}$	$\frac{1}{2}$
$\tan \theta$	$\frac{\sqrt{3}}{3}$	1	$\sqrt{3}$

Basic Trigonometric Identities

Reciprocal Identities

$$\begin{array}{lll} \csc \theta = \frac{1}{\sin \theta} & \sec \theta = \frac{1}{\cos \theta} & \cot \theta = \frac{1}{\tan \theta} \\ \sin \theta = \frac{1}{\csc \theta} & \cos \theta = \frac{1}{\sec \theta} & \tan \theta = \frac{1}{\cot \theta} \end{array}$$

Quotient Identities

$$\tan \theta = \frac{\sin \theta}{\cos \theta} \quad \cot \theta = \frac{\cos \theta}{\sin \theta}$$

Pythagorean Identities

$$\begin{aligned} \cos^2 \theta + \sin^2 \theta &= 1 \\ 1 + \tan^2 \theta &= \sec^2 \theta \\ \cot^2 \theta + 1 &= \csc^2 \theta \end{aligned}$$



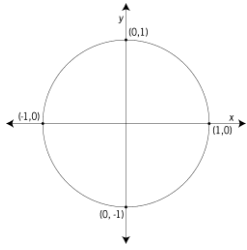
#1. Determine the sign (+ or -) of the following trigonometric functions.

- (a) $\tan 200^\circ$ _____ (b) $\tan 280^\circ$ _____
(c) $\cos 141^\circ$ _____ (d) $\cos 290^\circ$ _____
(e) $\sin \frac{4\pi}{3}$ _____ (f) $\sin \frac{5\pi}{6}$ _____
(g) $\cos(-\frac{\pi}{4})$ _____ (h) $\sin(-\frac{3\pi}{4})$ _____

#2.

(A) Sketch an angle that measures 150° in standard position.

Include an **arrow** to indicate the direction of the angle.



(B) What is the measure of the reference angle? _____

(C) $\sin 150^\circ =$ _____

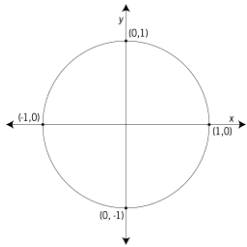
(D) $\cos 150^\circ =$ _____

(E) $\tan 150^\circ =$ _____

#3.

(A) Sketch an angle that measures $\frac{11\pi}{6}$ in standard position.

Include an **arrow** to indicate the direction of the angle.



(B) What is the measure of the reference angle? _____

(C) $\sin \frac{11\pi}{6} =$ _____

(D) $\cos \frac{11\pi}{6} =$ _____

(E) $\tan \frac{11\pi}{6} =$ _____



#4. State the value of each trigonometric function.

If the value is undefined, write "undefined".

(a) $\cos 270^\circ$ _____

(b) $\sin 180^\circ$ _____

(c) $\tan 90^\circ$ _____

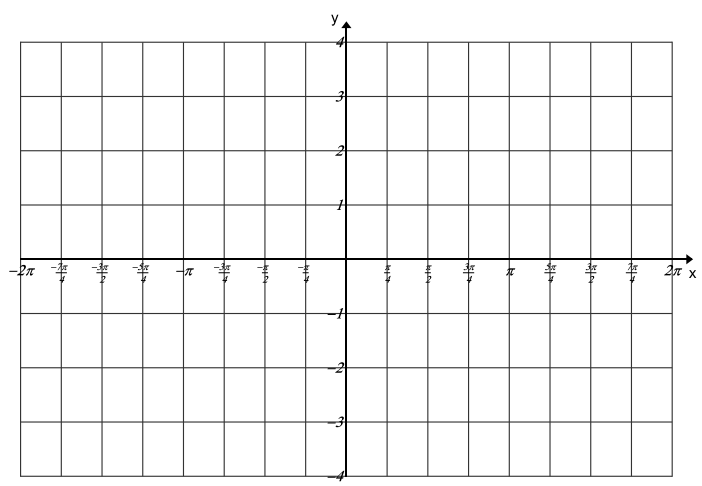
(d) $\cos 3\pi$ _____

(e) $\sin 5\pi$ _____

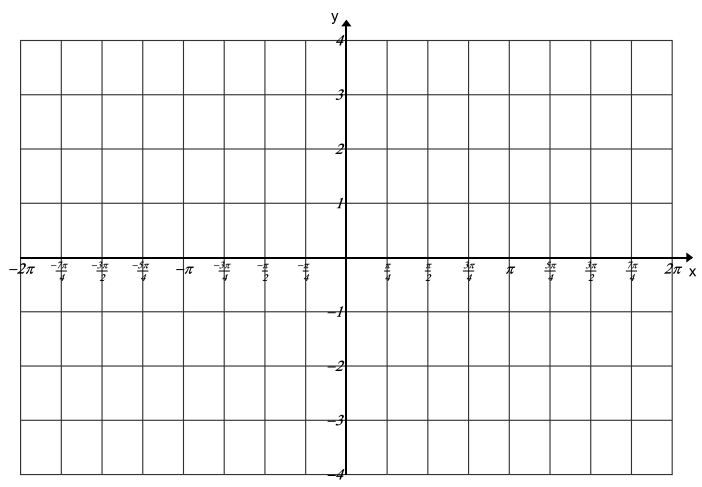
(f) $\tan \frac{3\pi}{2}$ _____

Graph one period of each function.

#5. $y = \sin x$

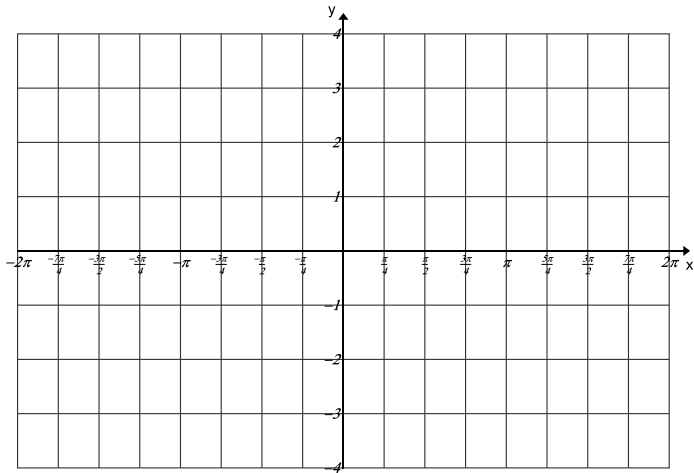


#6. $y = \cos x$

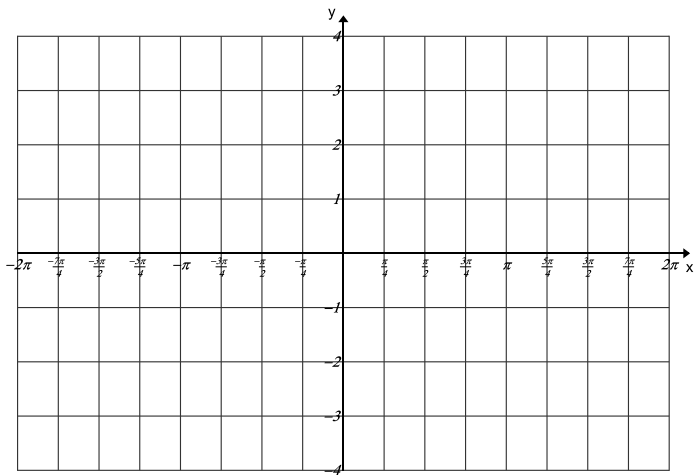


Graph one period of each function.

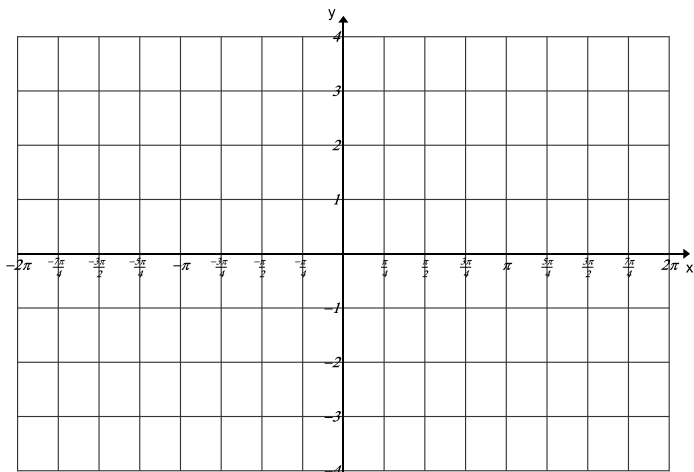
#7. $y = \tan x$



#8. $y = \cot x$

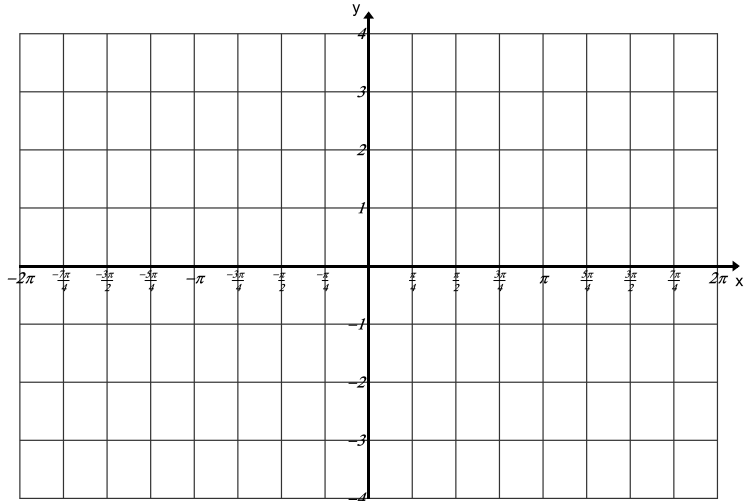


#9. $y = 4\cos x$

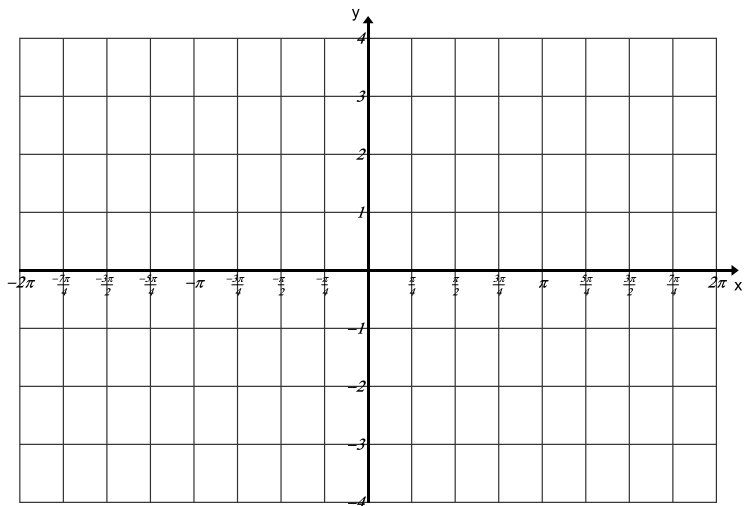


Graph one period of each function.

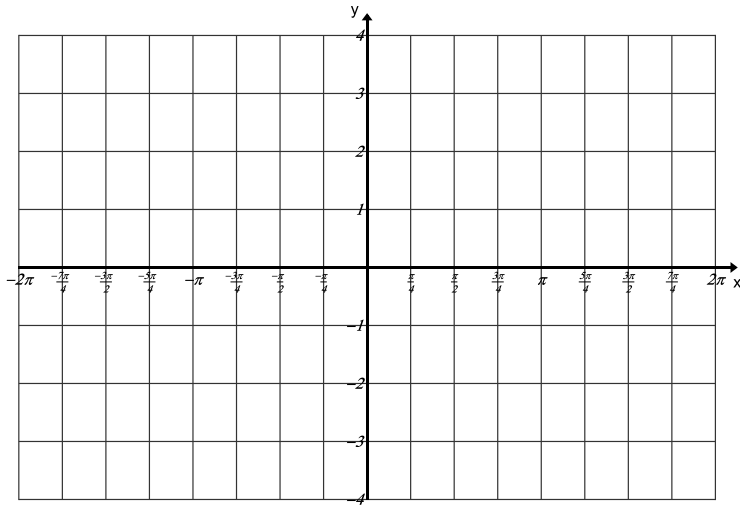
#10. $y = \cos 4x$



#11. $y = \sin x - 2$

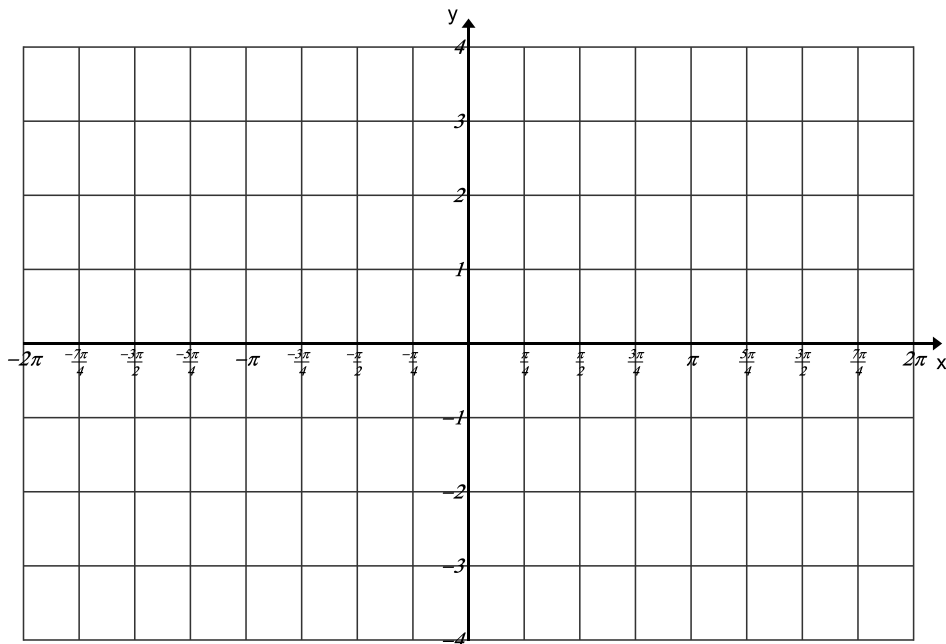


#12. $y = -\sin x$

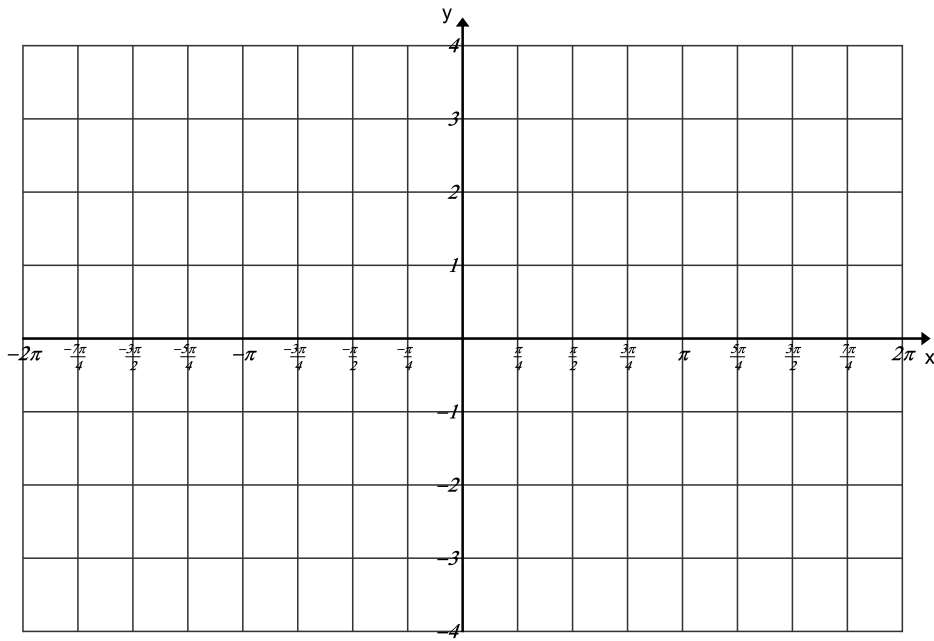


Graph one period of each function.

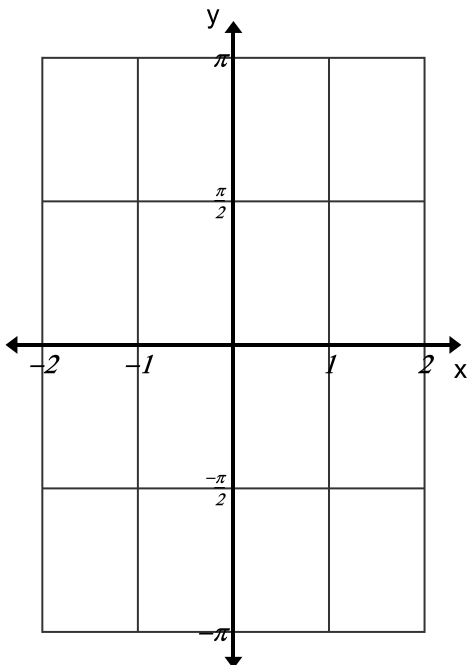
#13. $y = \sec x$



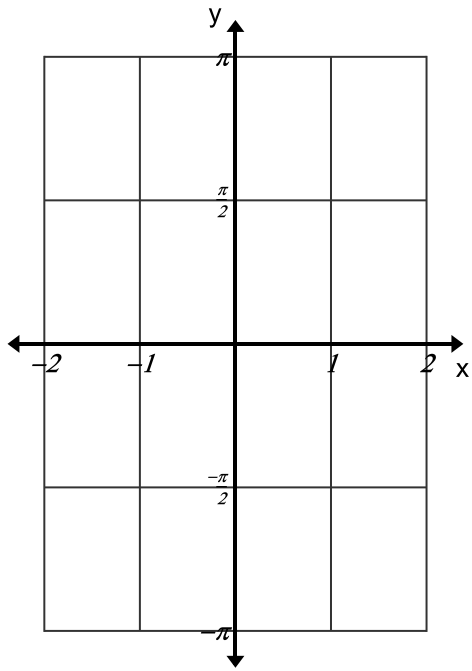
#14. $y = \csc x$



#15. Graph $\sin^{-1} x$



#16. Graph $\cos^{-1}x$





- #17 – 19.** (A) Find the **amplitude** and **period** of each function.
 (B) Describe how the graph of the function is related to the graph of $y = \sin x$ or $y = \cos x$.
 Use the language of transformations below.

LANGUAGE OF TRANSFORMATIONS

- **Vertical stretch/shrink by a factor of ____**
- **Horizontal stretch/shrink by a factor of ____**
- **Vertical shift up/down ____ units**
- **Horizontal shift right/left ____ units**
- **Reflects over the x-axis**
- **Reflects over the y-axis**

#17. $y = -\cos x$ (A) amplitude: ____ period: ____

(B) _____

#18. $y = 3\sin x - 6$ (A) amplitude: ____ period: ____

(B) _____

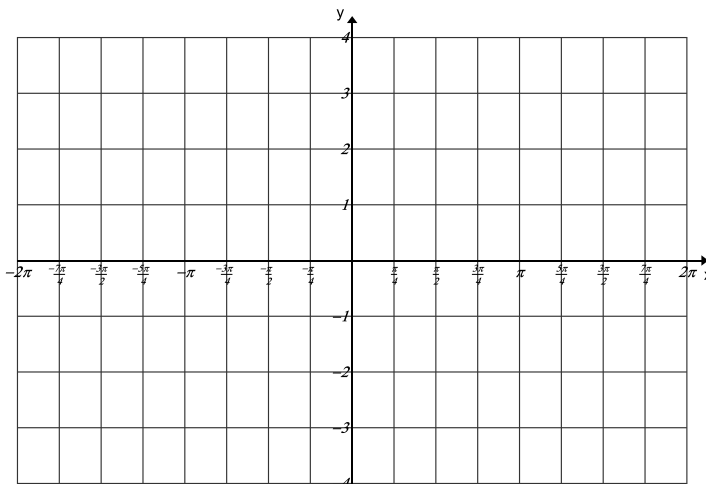
#19. $y = \cos \frac{1}{2}x$ (A) amplitude: ____ period: ____

(B) _____

#20. Graph one period of the function $y = 2 \sin \frac{1}{2}x + 1$

Domain: _____ Amplitude: _____

Period: _____ Range: _____

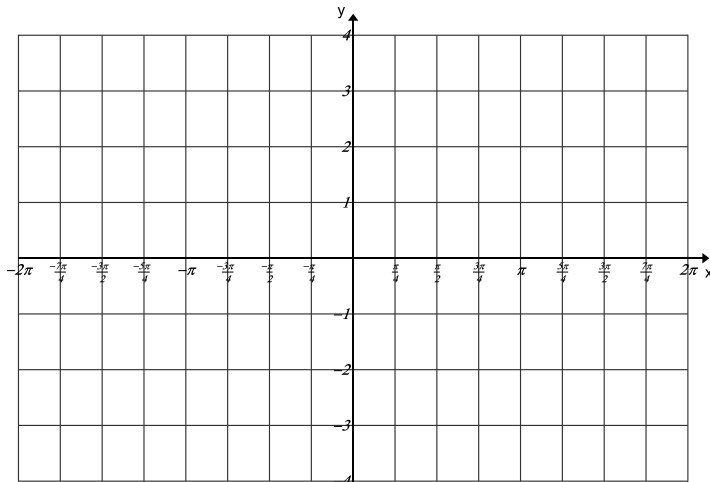




#21. Graph one period of the function $y = \sin(x - \pi)$

Domain: _____ Amplitude: _____

Period: _____ Range: _____



#22. – 27. Evaluate. Give your answer in RADIAN MEASURE.

#22. $\cos^{-1}\left(-\frac{1}{2}\right)$

#23. $\sin^{-1}\left(-\frac{\sqrt{2}}{2}\right)$

#24. $\cos^{-1}(0)$

#25. $\sin^{-1}(1)$

#26. $\tan^{-1}(1)$

#27. $\tan^{-1}(-1)$

Find the exact value of the expression, if possible.

#28. $\arctan\left(\tan\frac{5\pi}{3}\right)$

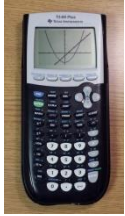
#29. $\cos^{-1}\left(\tan\frac{7\pi}{4}\right)$

#30. $\cos^{-1}\left(\cos\frac{3\pi}{2}\right)$

#31. $\sin\left(\cos^{-1}\left(-\frac{1}{2}\right)\right)$

Part 2

You may use a calculator.



#1. Multiple Choice.

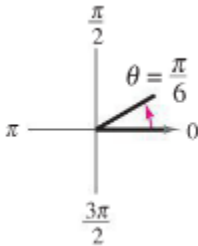
The measure of a central angle that intercepts an arc equal in length to the radius of the circle is _____.

- (a) one radian (b) π radians (c) one degree
(d) 90 degrees (e) none of these

#2. Convert 54° to radians. Give your answer **in terms of π** .
(Leave π in your answer.)

#3. Convert $\frac{5\pi}{9}$ radians to degrees.

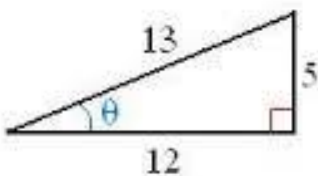
#4. Determine two coterminal angles in radian measure (one positive and one negative).



(A) positive: _____ (B) negative: _____

#5. Find the value of all six trigonometric functions of the angle θ .

Write your answers as fractions.



$\sin \theta =$ $\csc \theta =$

$\cos \theta =$ $\sec \theta =$

$\tan \theta =$ $\cot \theta =$

- #6. Assume that θ is an acute angle in a right triangle satisfying the given condition. Evaluate the cosine and tangent functions for angle θ . Answers should be in fraction form. Show all work.

$$\sin \theta = \frac{24}{25}$$

$$\cos \theta = ?$$

$$\tan \theta = ?$$

- #7. Evaluate using a calculator. Be sure your calculator is in the correct mode. Give answers correct to three decimal places.

(a) $\sin 72^\circ =$ _____

(b) $\cos 0.37 =$ _____

(c) $\cot \frac{\pi}{7} =$ _____

(d) $\sec 47^\circ =$ _____

(e) $\tan 25^\circ 24' =$ _____

- #8. Find the acute angle θ that satisfies the given equation. Give θ in DEGREE measure. Give answers correct to THREE decimal places.

(a) $\sin \theta = \frac{4}{5}$

(b) $\tan \theta = 3.9$

- #9. – 10. Simplify using trigonometric identities. Show your work.

#9. $\tan x \cos x$

#10. $\frac{1 - \sin^2 x}{\cos x}$

#11. - 12. Simplify using trigonometric identities.

#11. $\sin^2 x + \cot^2 x + \cos^2 x$

#12. $\sin x - \sin x \cos^2 x$

#13. Find all solutions to the equation in the interval $[0, 2\pi)$

$$2 \cos x + 1 = 0$$

#14. Find all solutions to the equation in the interval $[0, 360^\circ)$

$$\sin x + 1 = -\sin x$$

#15. Find all solutions to the equation in the interval $[0, 360^\circ)$

$$\tan^2 x - 1 = 0$$

#16. Find all solutions to the equation in the interval $[0, 2\pi)$

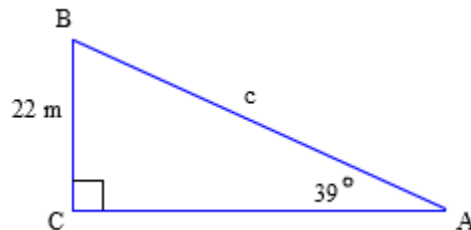
$$2 \cos x \sin x - \sin x = 0$$

#17. Find all solutions to the equation in the interval $[0, 2\pi)$

$$2 \cos^2 x + 3 \cos x + 1 = 0$$

#18. Round your answer to **THREE** decimal places as needed.

Find the measure of the side of the right triangle whose length is designated by the lower case letter c.



#19. Round your answer to THREE decimal places as needed.

From a point 360 feet away from the base of the Peachtree Center Plaza in Atlanta, Georgia, the angle of elevation to the top of the building is 61° . Find the height h of the building.



#20. Round your answer to three decimal places.

The angle of depression of a buoy from the top of a lighthouse 140 feet above the surface of the water is 7° . Find the distance x from the base of the lighthouse to the buoy.



#21. Round your answer to THREE decimal places as needed.

A backpacker notes that from a certain point on level ground, the angle of elevation to a point at the top of a tree is 38° . After walking 60 meters closer to the tree, the backpacker notes that the angle of elevation is 55° . Find the height of the tree.

#22. Round your answer to the nearest tenth.

Given: $\triangle ABC$ with $a = 8$, $b = 7$, and $\angle A = 26.5^\circ$
Use the Law of Sines to find the measure of angle B.

#23. Round your answer to three decimal places.

Given: $\triangle ABC$ with $a = 10$, $b = 6$, and $\angle C = 20^\circ$
Use the Law of Cosines to find the measure of side c .

#24. Round your answer to three decimal places.

Dana wants to find the distance between two points A and B on opposite sides of a building. She locates a point C that is 110 feet from A and 160 feet from B, as illustrated in the figure. If the angle at C is 54° , find the distance from A to B.

